Ion-acoustic waves in a dust-contaminated plasma

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A self-consistent theory of ion-acoustic waves in dusty gas discharge plasmas is presented. The plasma is contaminated by fine dust particles with variable charge. The stationary state of the plasma and the dispersion and damping characteristics of the waves are investigated accounting for ionization, recombination, dust charge relaxation, and dissipation due to electron and ion elastic collisions with neutrals and dusts, as well as charging collisions with the dusts. [S1063-651X(99)02609-4]

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I. INTRODUCTION

Massive negatively charged grains are found in many space plasmas as well as industrial plasma-processing chambers [1–3]. The highly charged [4] dusts can significantly affect collective plasma processes by modifying the overall charge balance and introducing charge relaxation which leads to dust-charge fluctuations [5,6]. The effect of dusts on acoustic waves in space and laboratory plasmas was studied by many authors [7–12]. However, most theoretical models consider dust-charge variation without properly taking into account the change of the plasma particle number density arising from their capture (and release) by the dust grains. Unspecified sources and sinks are sometimes invoked but not self-consistently included in the conservation equations, assuming that their total effect is small. On the other hand, sinks of plasma electrons and ions resulting from their capture by the dusts can be accounted for by considering the dusty plasma as thermodynamically open and introducing the corresponding capture processes into the electron and ion conservation equations [6]. However, merely accounting for the plasma particle capture processes is not sufficient, since without a balancing source for the plasma particles there cannot exist a self-consistent stationary state. It is therefore necessary to take into account the creation of plasma particles by, say, ionization. For most plasmas it is then also necessary to include related plasma-particle loss and transport mechanisms such as ambipolar diffusion and volume recombination, which usually take place at the same time scale as that of ionization. Since ionization, diffusion, and recombination are all density dependent, electrostatic processes in the system can be strongly affected. This opens the possibility that the presence of dusts can affect the entire discharge system through a modification of an ionization-recombination-diffusion balance.

The effect of variable-charge dust grains on Langmuir waves in a low-temperature gas-discharge plasma was investigated recently [13]. It was shown that because of similar time scale and order of magnitude of their contributions, the dust-charge relaxation process is closely associated with the ionization and recombination processes in the plasma. Together they self-consistently maintain the background particle number densities during the perturbations, and the overall ionization-recombination balance allows for the existence of the equilibrium state of the discharge. It was demonstrated that dissipation due to electron collisions with ions, neutrals, or other electrons, as well as elastic (Coulomb) and inelastic (dust-charging) collisions with dust grains, leads to a net damping of the Langmuir waves in typical laboratory dusty plasmas [13], in contrast to the instability [10] obtained by ignoring the sources and sinks in the electron and ion conservation equations.

For the relatively high frequency Langmuir waves the ion motion is unimportant [13]. However, for the lower-frequency ion-acoustic waves (IAWs), a self-consistent theory should include a proper account of the ions. In the present paper, the effect of variable-charge dusts on the propagation and damping of the ion-acoustic waves in gas discharge plasmas is investigated self-consistently by including both electron and ion subsystems with elastic and charging interactions and taking into account the overall particle balance.

The paper is organized as follows. In Sec. II, we formulate the problem and present the basic set of equations. In Sec. III, the stationary electron and ion densities are calculated and the dispersion relation of the ion-acoustic waves is derived. Dispersion and damping of the IAWs are studied in Sec. IV. In Sec. V the validity of the obtained results is discussed.

II. FORMULATION

We consider the linear propagation of ion-acoustic waves in a plasma with finite electron (T_e) and ion (T_i) temperatures. The size of the dust grains is assumed to be much less

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than the intergrain distance, the electron Debye radius, as well as the wavelength, so that they can be treated as heavy point masses. The charge of a dust grain varies because of the microscopic electron and ion currents flowing into it according to the potential difference between its surface and the adjacent plasma. The dusts are treated as an immobile background since the time scale of the charge variation is much smaller than that of the dust motion [6].

The equations for describing ion-acoustic waves in a collisional plasma with variable-charge dust grains are given by

$$\partial_t n_e + \nabla\cdot(n_e v_e) = -\nu_{ed} n_e + \nu_{\rm ion} n_e - \beta_{\rm eff} n_e^2, \tag{1}$$

$$\partial_t v_e + \nu_e^{\rm eff} v_e + \frac{T_e}{m_e n_e}\partial_x n_e = -\frac{e}{m_e}E, \tag{2}$$

$$\partial_t n_i + \nabla\cdot(n_i v_i) = -\nu_{id} n_i + \nu_{\rm ion} n_e - \beta_{\rm eff} n_e^2, \tag{3}$$

$$\partial_t v_i + \nu_i^{\rm eff} v_i + \frac{T_i}{m_i n_i}\partial_x n_i = \frac{e}{m_e}E, \tag{4}$$

$$d_t \tilde{q}_d + \nu_d^{\rm ch}\tilde{q}_d = -|I_{e0}|\tilde{n}_e/n_{e0} + |I_{i0}|\tilde{n}_i/n_{i0}, \tag{5}$$

and

$$\nabla^2\varphi = -4\pi e(n_i - n_e - Z_d n_d), \tag{6}$$

where $E=-\nabla\varphi$ is the electric field of the ion-acoustic waves, φ is the electrostatic potential, m_j, $n_j = n_{j0}+\tilde{n}_j$, and v_j are the mass, density, and fluid velocity of the species $j = e,i,d$ for electron, ion, and dust, respectively. Furthermore, $q_d = -Z_d e = q_{d0}+\tilde{q}_d$ is the average charge of the dust particles.

In Eqs. (1) and (3), $\nu_{\rm ion}$ is the ionization rate, $\beta_{\rm eff}=\beta-\beta_{\rm si}$, β is the volume recombination rate [14], and $\beta_{\rm si}$ is the stepwise ionization rate. In the dust-charging equation (5), $\nu_d^{\rm ch} = a\omega_{pi}^2\mathcal{A}/\sqrt{2\pi}V_{Ti}$ is the dust charging rate [6], where $\mathcal{A} = 1+(T_i/T_e)+(\varphi_d^{\rm el}/\varphi_e^{\rm th})$, $\varphi_d^{\rm el} = Z_d e/a$, $\varphi_e^{\rm th} = T_e/e$, $V_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal velocity, ω_{pi} is the ion plasma frequency, and a is the average dust radius. We have also defined in Eq. (2) the effective frequency of electron collisions as $\nu_e^{\rm eff} = \nu_e + \nu_e^{\rm el} + \nu_e^{\rm ch}$, and in Eq. (4) that of ion collisions as $\nu_i^{\rm eff} = \nu_i + \nu_i^{\rm el} + \nu_i^{\rm ch}$. Furthermore, $\nu_e = \nu_{en}+\nu_{ei}$ and $\nu_i = \nu_{in}+\nu_{ie}$ are the rates of electron and ion collisions with the neutrals and plasma particles; for the charging collision frequency we have [6,15]

$$\nu_e^{\rm ch} = \frac{3}{2}\nu_i^{\rm ch}\frac{n_{i0}}{n_{e0}}\frac{\alpha\sigma}{T_i/T_e+\alpha} = \nu_d^{\rm ch}\frac{\alpha\gamma\sigma}{\mathcal{A}}, \tag{7}$$

while the rate of electron and ion capture by the grain is given by

$$\nu_{ed} = \frac{n_{i0}}{n_{e0}}\nu_{id} = \nu_d^{\rm ch}\frac{\alpha\gamma}{\mathcal{A}}, \tag{8}$$

where $\alpha = \mathcal{A}-1$, $\gamma = (Z_{d0}n_{d0}/n_{e0})\varphi_e^{\rm th}/\varphi_d^{\rm el}$, and $\sigma = 4+\varphi_d^{\rm el}/\varphi_e^{\rm th}$. The rates of elastic electron- and ion-dust Coulomb collisions are

$$\nu_e^{\rm el} = \nu_i^{\rm el}\frac{T_i}{T_e}\frac{n_{i0}}{n_{e0}}\alpha\exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right) = \frac{2}{3}\nu_d^{\rm ch}\frac{\alpha\gamma\Lambda}{\mathcal{A}}\exp\left(\frac{\varphi_d^{\rm el}}{\varphi_e^{\rm th}}\right), \tag{9}$$

where Λ is the Coulomb logarithm. The electron- and ion-neutral collision frequencies are $\nu_{en} = N_n\sigma_{en}V_{Te}$, $\nu_{in} = N_n\sigma_{in}V_{Ti}$, where N_n is the neutral gas density, σ_{en} and σ_{in} are the electron (ion) -neutral collision cross sections, and V_{Te} and V_{Ti} are electron and ion thermal velocities [16]. The expressions for the electron-ion and ion-electron collision frequencies can easily be found in [17]. For simplicity, we consider intermediate pressures so that recombination loss prevails over that of diffusion. The electron-ion collisional (also ionization and recombination) terms are standard. The collision terms involving dusts are also similar. Those involving dust-charge variation have been calculated using the kinetic theory [6,15]. The fact that the dissipative loss in dusty plasmas is generally higher than that in dust-free plasmas (due to Coulomb collisions with and capture by the dusts) has been noticed experimentally [8,18].

In the absence of perturbations the system is charge neutral, $n_{i0} = n_{e0}+Z_{d0}n_{d0}$, where the stationary dust charge is defined by the equality of equilibrium microscopic electron and ion currents flowing onto the dust particles due to the local potential difference between the dust and the adjacent plasma,

$$I_{e0} = -\pi a^2 e(8T_e/\pi m_e)^{1/2} n_{e0}\exp(e\Delta\varphi_g/T_e), \tag{10}$$

$$I_{i0} = \pi a^2 e(8T_i/\pi m_i)^{1/2} n_{i0}(1-e\Delta\varphi_g/T_i), \tag{11}$$

where $\Delta\varphi_g$ is the steady-state potential difference between the dust grain and the adjacent plasma.

III. DISPERSION RELATION

From Eq. (1) we obtain

$$n_{e0} = (\nu_{\rm ion}-\nu_{ed})/\beta_{\rm eff} \tag{12}$$

for the stationary electron density in the dust-contaminated discharge. It follows from Eq. (12) that the rate of ionization must exceed that of electron capture by the dusts. Physically, electron loss to the dusts is compensated by an additional ionization. Thus, the dusts strongly affect the particle balance in the discharge. The stationary state is maintained by a balance between ionization, recombination, and plasma particle capture by the dusts. Substituting Eq. (12) in Eq. (3), we obtain $n_{i0} = (\nu_{ed}/\nu_{id})n_{e0}$ for the equilibrium ion density. We see that the equilibrium electron and ion densities are different and are inversely proportional to the rates of electron and ion capture by dust. The same result follows from Eq. (8) and the overall charge neutrality condition. It is easy to verify that if one had used the ion density n_i instead of the electron density n_e in the ionization and recombination terms on the right-hand side of Eq. (3), one would obtain an incorrect steady state not consistent with the charge-neutrality condition.

Assuming that perturbation quantities behave like $\exp[i(kz-\omega t)]$, we linearize Eqs. (1)–(6), taking into account Eq. (12), and obtain the dust charge perturbation

$$\tilde{Z}_d = -i\frac{|I_{e0}|}{en_{e0}(\omega+i\tilde{\nu}_d^{\mathrm{ch}})}\left(\frac{k^2\varphi}{4\pi}-\frac{Z_{d0}n_{d0}}{n_{i0}}\tilde{n}_i\right), \tag{13}$$

where $\tilde{\nu}_d^{\mathrm{ch}}=\nu_d^{\mathrm{ch}}+\nu_{ed}$, the electron fluid velocity

$$v_e=-\frac{kT_e}{m_e(\omega+i\nu_e^{\mathrm{eff}})}\left(\frac{e\varphi}{T_e}-\frac{\tilde{n}_e}{n_{e0}}\right), \tag{14}$$

the perturbed electron density

$$\tilde{n}_e=-\frac{ek^2n_{e0}\varphi}{\eta_e m_e(\omega+i\nu_e^{\mathrm{eff}})}, \tag{15}$$

where $\eta_e=\omega+i(\nu_{\mathrm{ion}}-\nu_{ed})-k^2V_{Te}^2/(\omega+i\nu_e^{\mathrm{eff}})$ and $V_{Te}=(T_e/m_e)^{1/2}$ is the electron thermal velocity, the ion fluid velocity

$$v_i=\frac{kT_i}{m_i(\omega+i\nu_i^{\mathrm{eff}})}\left(\frac{e\varphi}{T_i}+\frac{\tilde{n}_i}{n_{i0}}\right), \tag{16}$$

and the perturbed ion density

$$\tilde{n}_i=\frac{ek^2n_{i0}\varphi}{\eta_i m_i(\omega+i\nu_e^{\mathrm{eff}})}(1-i\mathcal{G}), \tag{17}$$

where

$$\mathcal{G}=\frac{n_{e0}}{n_{i0}}\frac{m_i}{m_e}\frac{\omega+i\nu_i^{\mathrm{eff}}}{\omega+i\nu_e^{\mathrm{eff}}}\frac{2\nu_{ed}-\nu_{\mathrm{ion}}}{\eta_e}$$

and $\eta_i=\omega+i\nu_{id}-k^2V_{Ti}^2/(\omega+i\nu_i^{\mathrm{eff}})$. For simplicity, we have neglected the variation of the rates of electron and ion capture with the variation of the dust charge. The conditions for this assumption will be discussed later.

Substitution of Eqs. (13), (15), and (17) into Eq. (6) leads to

$$1-i\frac{\nu_{ed}}{\omega+i\tilde{\nu}_d^{\mathrm{ch}}}-\frac{\omega_{pe}^2}{\eta_e(\omega+i\nu_e^{\mathrm{eff}})}-\frac{\omega_{pi}^2}{\eta_i(\omega+i\nu_i^{\mathrm{eff}})}\times\left(1-i\frac{Z_dn_{d0}}{n_{i0}}\frac{\nu_{ed}}{\omega+i\tilde{\nu}_d^{\mathrm{ch}}}\right)(1-i\mathcal{G})=0, \tag{18}$$

which is the dispersion relation for the ion-acoustic waves in a collisional dusty plasma. In the absence of dissipation and dust-charge variation we have from Eq. (18) $\omega=kV_{SD}$, where $V_{SD}=\sqrt{(T_e/m_i)(n_{i0}/n_{e0})+T_i/m_i}$ is the ion-acoustic speed in a dissipationless plasma with constant-charge dust. If $k^2V_{Te}^2\gg[\omega^2,\nu_e^{\mathrm{eff}}(\nu_{\mathrm{ion}}-\nu_{id})]$, which is usually valid for ion-acoustic waves, from Eq. (18) we obtain

$$(\omega+i\tilde{\nu}_d^{\mathrm{ch}})\{(\omega+i\nu_i^{\mathrm{eff}})(\omega+i\nu^\star)-k^2V_{SD}^2\} = -i\frac{\nu_{ed}Z_dn_{d0}}{n_{i0}}[k^2(V_{SD}^2-V_{Ti}^2) + i(2\nu_{ed}-\nu_{\mathrm{ion}})(\omega+i\nu_i^{\mathrm{eff}})], \tag{19}$$

where $\nu^\star=\nu_{id}+\nu_{\mathrm{ion}}-2\nu_{ed}$. If we retain in Eq. (19) dust-charging terms and usual electron- and ion-neutral collisions only, we obtain the coupling equation similar to that for ion-acoustic surface waves [12].

IV. PROPAGATION AND DAMPING OF ION-ACOUSTIC WAVES

To solve the dispersion relation (19), we assume a real wave frequency and obtain a complex wave number [18]. After lengthy but straightforward algebra, we obtain $k=k'+ik''$, where

$$k'=\zeta/\sqrt{2}V_{SD} \quad \text{and} \quad k''=\Theta''/(\sqrt{2}V_{SD}\zeta), \tag{20}$$

where

$$\zeta=\{\Theta'+[(\Theta')^2+(\Theta'')^2]^{1/2}\}^{1/2},$$

$$\Theta'=(\delta_1\delta_2-\delta_3\delta_4)/W,$$

$$\Theta''=(\delta_1\delta_4+\delta_2\delta_3)/W,$$

$$W=\omega^2+(\tilde{\nu}_d^{\mathrm{ch}}+\vartheta_1)^2,$$

$$\vartheta_1=(\nu_{ed}Z_dn_{d0}/n_{i0}V_{SD}^2)(V_{SD}^2-V_{Ti}^2),$$

$$\delta_1=\omega^2+\nu_i^{\mathrm{eff}}(\tilde{\nu}_d^{\mathrm{ch}}+\vartheta_1),$$

$$\delta_2=\omega^2-\nu_{id}\tilde{\nu}_d^{\mathrm{ch}}+(\nu_{\mathrm{ion}}-2\nu_{ed})(-\tilde{\nu}_d^{\mathrm{ch}}+\nu_{ed}Z_dn_{d0}/n_{i0}),$$

$$\delta_3=-\omega(\tilde{\nu}_d^{\mathrm{ch}}+\vartheta_1)+\nu_i^{\mathrm{eff}},$$

and $\delta_4=\omega(\nu^\star+\tilde{\nu}_d^{\mathrm{ch}})$. This solution is valid when the frequency of the ion-acoustic waves is of the order of or even less than the rate capture of the plasma electrons by the dust grains.

For high frequencies ($\omega\gg\nu_{ed}Z_dn_{d0}/n_{i0}$), the relation (19) describes the coupling of the ion-acoustic waves and the dust-charge relaxation process. We have then two equations, namely $D_1(\omega,k)=(\omega+i\nu_i^{\mathrm{eff}})(\omega+i\nu^\star)-k^2V_{SD}^2=0$ for the ion-acoustic waves and $D_2(\omega,k)=\omega+i\tilde{\nu}_d^{\mathrm{ch}}=0$ for the dust-charge relaxation process. The solution of the dispersion equation $D_1(\omega,k)=0$ is

$$\omega=-i\frac{\nu_i^{\mathrm{eff}}+\nu^\star}{2}\pm\left[k^2V_{SD}^2-\frac{(\nu_i^{\mathrm{eff}}+\nu^\star)^2}{4}\right]^{1/2}, \tag{21}$$

which describes damped ion-acoustic waves in a low-temperature dusty plasma. From Eq. (21) it follows that propagating waves exist if $kV_{SD}>(\nu_i^{\mathrm{eff}}+\nu^\star)/2$. The real and imaginary parts of the frequency are $\omega_{IA}'=[k^2V_{SD}^2-(\nu_i^{\mathrm{eff}}+\nu^\star)^2/4]^{1/2}$ and $\omega_{\mathrm{IA}}''=-(\nu_i^{\mathrm{eff}}+\nu^\star)/2$. However, if $kV_{SD}<(\nu_i^{\mathrm{eff}}+\nu^\star)/2$, the solution is purely damped. The coupling with the dust-charge relaxation process leads to the variation $\delta\omega_{\mathrm{IA}}$ of the IAW eigenfrequency,

$$\delta\omega_{\mathrm{IA}}=-i\frac{\nu_{ed}}{\omega_{\mathrm{IA}}+i\tilde{\nu}_d^{\mathrm{ch}}}\frac{Z_{d0}n_d}{n_{i0}}\frac{\mathcal{E}}{2\omega_{\mathrm{IA}}'}, \tag{22}$$

where $\mathcal{E}=k^2(V_{SD}^2-V_{Ti}^2)+i(2\nu_{ed}-\nu_{\rm ion})(\omega_{\rm IA}+i\nu_i^{\rm eff})$. From Eq. (22) we see that the coupling of IAWs to the dust-charge relaxation process leads to a frequency downshift and an increase of the damping decrement.

The corresponding solution for the dust-charge relaxation process is $\omega_D=-i\tilde{\nu}_d^{\rm ch}+\delta\omega_D$, where

$$\delta\omega_D=i\nu_{ed}\frac{Z_{d0}n_d}{n_{i0}}\frac{\mathcal{B}}{\mathcal{D}},$$

where

$$\mathcal{B}=k^2(V_{SD}^2-V_{Ti}^2)+(2\nu_{ed}-\nu_{\rm ion})(\nu_i^{\rm eff}-\tilde{\nu}_d^{\rm ch})$$

and $\mathcal{D}=k^2V_{SD}^2+(\nu_i^{\rm eff}-\tilde{\nu}_d^{\rm ch})(\nu^\star-\tilde{\nu}_d^{\rm ch})$. We note that the dust-charge relaxation rate is reduced due to coupling with the ion-acoustic waves.

Expressions (20)–(22) describe the propagation of ion-acoustic waves in a plasma contaminated by variable-charge dusts for a wide range of parameters of interest. The frequency of the IAWs should not be too low, otherwise inclusion of temperature fluctuations and use of the full Braginski's equations [19] for strongly collisional plasmas would be necessary [20]. This is especially true if the frequency of IAWs is less than the effective rate of ion collisions.

V. DISCUSSION AND CONCLUSION

It should be noted that the volume recombination rate β can also depend on ion density [14]. This is because the electron-ion recombination usually goes through three-body collisions (when the ionization degree and n_e, n_i are not too low) and the steplike nature of the latter process. This, however, does not affect the validity of our results, since the perturbation (not included) of the recombination coefficient β would be proportional to $\tilde{n}_i$, which is much less than $\tilde{n}_e$ since $m_i \gg m_e$ and $\eta_e \sim \eta_i$. However, care should be taken such that n_{e0} does not become much less than n_{i0} and comparable to $(m_e/m_i)n_{i0}$. Otherwise the perturbation of the recombination coefficient may have to be included.

We have also neglected the fluctuations of the electron and ion capture frequencies arising from the variation of the average dust charge. This assumption is valid if the conditions

$$\partial_{Z_d}\nu_{(e,i)d} \ll \nu_{(e,i)d} \tag{23}$$

are fulfilled. The largest value of the ratio $\partial_{Z_d}\nu_{(e)d}/\nu_{(e)d}$ is approximately 0.125, so that the conditions (23) are indeed satisfied. However, for larger (10 μm or more) dust particles the conditions could be violated.

In our investigation the density of the neutral gas N_n enters the expressions for the electron-neutral and ion-neutral collisions as well as the ionization rates. However, the variation of N_n is only due to ionization and recombination, since the neutrals do not respond to the electrostatic waves. One can easily include the fluctuation of N_n (due to ionization and recombination) in ν_{en}, ν_{in}, and $\nu_{\rm ion}$. But since our plasma is weakly ionized, and the actual perturbations are of the order of $\tilde{n}_e$, the perturbed quantities $\tilde{N}_n/N_{n0}$ will be much smaller than $\tilde{n}_e/n_{e0}$. Thus the change of N_n does not contribute significantly to the wave motion.

It is also necessary to satisfy the condition for the existence of a stationary state of the dusty discharge. That is, the ionization rate must exceed the rate of electron capture by the dust grains, assuming that there is no secondary emission or surface ionization of the dust. It can be shown that the inequality $\nu_{\rm ion}>\nu_{ed}$ is realized for typical (say, hydrogen) plasmas with $n_{i0}\sim 10^{10}$ cm^{-3}, $N_n\sim 5\times 10^{14}$ cm^{-3}, $a \sim 1$ μm, $n_{i0}/n_{e0}=2$, and $\varphi_d^{\rm el}/\varphi_e^{\rm th}\sim 2$. The validity of the inequality $\nu_{\rm ion}>\nu_{ed}$ strongly depends on the electron temperature. We find that the threshold value $T_e^{\rm thres}$ defined by the condition $\nu_{\rm ion}=\nu_{ed}$ is about 1.5 eV. For argon and potassium plasmas, similar results are expected, although the corresponding expressions for the ionization frequencies depend on several factors and are thus more complicated.

Finally, we discuss the applicability of our results to specific gas discharge plasmas. For argon ($m_i/m_e\sim 73\,600$) plasmas at intermediate pressure (100 mTorr) with $T_e \sim 1$ eV, $T_i\sim 0.1$ eV, and $N_n\sim 5\times 10^{15}$ cm^{-3}, assuming $\sigma_{en}\sim 1.5\times 10^{-16}$ cm^{-3}, and $\sigma_{in}\sim 3\times 10^{-15}$ cm^{-3} [21], we obtain $\nu_{en}\sim 1.8\times 10^7$ s^{-1} and $\nu_{in}\sim 4.2\times 10^5$ s^{-1}. Assuming $n_{0i}\sim 10^{10}$ cm^{-3} and $n_{0e}\sim 0.5n_{0i}$, one obtains for the electron-ion and ion-electron collision frequencies $\nu_{ei} \sim 10^5$ s^{-1} and $\nu_{ie}\sim 2\times 10^4$ s^{-1}. Thus, our results are valid for operating frequencies of the order of 5×10^4 s^{-1} and above, otherwise an approach similar to that in [20] would be required. Furthermore, care should be exercised in using the compact relation (19) because of the condition $kV_{Te} \gg (\omega,\nu')$, where ν' is a dissipation rate such as $\tilde{\nu}_d^{\rm ch}$, $\nu_i^{\rm eff}$, ν_{id}, or ν_{ed}. For example, for the experiment on dust ion-acoustic waves by Barkan *et al.* [22], we have $m_i/m_e \sim 71\,870$ (potassium plasma), $T_i\approx T_e\approx 0.2$ eV, $a\sim 5$ μm, $r_{De}\sim 0.3$ cm, $n_{i0}\approx 10^{10}$ cm^{-3}, $n_{d0}\approx 10^5$ cm^{-3}, and $n_{e0}/n_{i0}\approx 0.4$. The ratio $\varphi_d^{\rm el}/\varphi_e^{\rm th}$ can be calculated from

$$\sqrt{T_i/T_e}\exp(-\varphi_d^{\rm el}/\varphi_e^{\rm th})=(n_{i0}/n_{e0})\alpha\sqrt{m_e/m_i},$$

which follows from the balance of the electron and ion currents flowing onto the dust grain [6]. We obtain $\varphi_d^{\rm el}/\varphi_e^{\rm th} \approx 3.23$. It is easy to show that for the generator frequency 50 kHz with the wavelength ~ 10 cm [22], one has $\omega\approx 3.14 \times 10^5$ sec^{-1}, $\tilde{\nu}_d^{\rm ch}\sim 5\times 10^7$ sec^{-1}, and $kV_{Te}=1.1 \times 10^7$ sec^{-1}. Thus, the condition $kV_{Te}\gg(\nu',\omega)$ is not satisfied and the general dispersion equation (18) should be used.

To conclude, we have presented a self-consistent theory of ion-acoustic waves in a plasma with variable-charge dust particles. The theory accounts for the variation of the electron and ion number densities as well as the average dust charge. Elastic and charging collisions of the plasma electrons and ions with the dust grains are also included. Accounting for the ionization and recombination processes also leads to a self-consistent determination of the equilibrium state of the dust-contaminated discharge.

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